Distributed Resource Allocation in Fog Networks

Xiliang Luo

School of Information Science & Technology
ShanghaiTech University
Background

Wireless data traffic is increasing at an exponential rate

Solution: Densification

Higher Spectral Efficiency

More Spectrum

unlicensed
Solution: Densification

Fog-like Heterogeneous Networks:
Dense Small Cells $\rightarrow$ splitting gains
Problem Statement

- Conventional Resource Allocation/User Association in HetNets:
  - Max-DL-SINR association as in 2G/3G/4G networks
  - Cell Range Expansion [Sesia et al’11, Andrews et al’14]
  - Joint optimization of association and resource allocation
    - Assuming Best-Effort (BE) traffic
  - Performance metrics
    - the sum of log-rate [Fooladivanda et al’13, Ye et al’13, Deb et al’14, Kuang’16]
    - number of admitted users [Li et al’12]
    - sum of the inverse of the per-user throughput [Chen’11]
    - average packet delay [Zhang’16, Zhuang’15]
Problem Statement

- Remaining problems:
  - Most works did not take into account the QoS requirements explicitly
  - In an IP network, one metric mostly used is the IP packet transfer delay
  - Centralized solver is required to obtain the solution

- Our goal:
  - Model the DL QoS traffic explicitly
  - Optimize the network-wide packet delay performance
  - Distributed solution
HetNet with QoS Traffic

**Definitions:**
- $B$: system BW in Hz
- $P_n$: txmn pwr of BS-n
- $\lambda_k$: pkt arrival rate of MS-k
- $L_k$: avg pkt length in bits of MS-k
- $h_{k,n}$: channel btw MS-k and BS-n
- $x_{k,n}$: assoc indicator of MS-k to BS-n
- $y_{k,n}$: resource allocation

**Assumptions:**
- i.i.d. exp inter-arrival times
- i.i.d. exp packet lengths
- BS always ON
- full freq. reuse
Problem Formulation

- Avg rate of MS-k associated with BS-n occupying all resources:

\[ R_{k,n} = B \log \left( 1 + \frac{P_n |h_{k,n}|^2}{\sum_{l=1, l \neq n}^{N} P_l |h_{k,l}|^2 + \sigma^2} \right) \]

- Service time for the traffic towards MS-k is i.i.d. exp with mean:

\[ t_{k,n} = \frac{L_k}{y_{k,n} R_{k,n}} = \frac{1}{y_{k,n} r_{k,n}} \]

- Avg delay for a packet in the M/M/1 queue:

\[ \tau_{k,n} = \frac{1}{1/t_{k,n} - \lambda_k} = \frac{1}{y_{k,n} r_{k,n} - \lambda_k} \]
Problem Formulation

- Optimal association rule to minimize the average pkt delay across the whole network:

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{\sum_{k=1}^{K} \lambda_k} \sum_{n=1}^{N} \sum_{k=1}^{K} x_{k,n} \lambda_k \cdot F \left( \tau_{k,n} \right) w_{k,n} \\
\text{subject to} & \quad \sum_{n=1}^{N} x_{k,n} = 1.0, \quad \forall k = 1, \ldots, K \\
& \quad \sum_{k=1}^{K} y_{k,n} \leq 1.0, \quad \forall n = 1, \ldots, N \\
& \quad x_{k,n} \in \{0, 1\}, \quad \forall k = 1, \ldots, K, \quad \forall n = 1, \ldots, N \\
& \quad y_{k,n} > \frac{x_{k,n} \lambda_k}{r_{k,n}}, \quad \forall k = 1, \ldots, K, \quad \forall n = 1, \ldots, N.
\end{align*}
\]

- A classic knapsack problem is NP-hard [Bertsekas’99]
- We will need to find low-complexity approx. solutions!
Proposition 1:

At BS-n, given a feasible user association, i.e. the following condition is met:

\[
\sum_{k=1}^{K} \frac{x_{k,n} \lambda_k}{r_{k,n}} < 1.0,
\]

for \( F(\tau_{k,n}) = \tau_{k,n} \), \( w_{k,n} = x_{k,n} \lambda_k / r_{k,n} \), the optimal resource allocation minimizing the average packet delay is as follows:

\[
y_{k,n} = \frac{1}{\sum_{u=1}^{K} \frac{x_{u,n} \lambda_u}{r_{u,n}}} \frac{x_{k,n} \lambda_k}{r_{k,n}}.
\]
Simplified Problem

Optimal Association Problem:

Relax the integer association constraint in problem P1!

\[ P_2 \]

\[
\text{minimize } f(\{x_{k,n}\}) := \sum_{n=1}^{N} f_n(\{x_{k,n}\})
\]

subject to

\[
\sum_{n=1}^{N} x_{k,n} = 1, \quad \forall k = 1, \ldots, K
\]

\[
x_{k,n} \geq 0, \quad \forall k = 1, \ldots, K, \quad n = 1, \ldots, N
\]

\[
\sum_{k=1}^{K} \frac{x_{k,n} \lambda_k}{r_{k,n}} < 1, \quad \forall n = 1, \ldots, N,
\]

\[
f_n(\{x_{k,n}\}) = \left( \sum_{k=1}^{K} \frac{x_{k,n} \lambda_k}{r_{k,n}} \right)^2 = \frac{1}{\rho_{k,n} - \frac{x_{k,n} \lambda_k}{r_{k,n}}} + \rho_{k,n} - \frac{x_{k,n} \lambda_k}{r_{k,n}} - 2,
\]

\[
\rho_{k,n} := 1 - \sum_{u=1}^{K} x_{u,n} \frac{\lambda_u}{r_{u,n}},
\]
Proposition 2

For MS-k, given others’ association, the optimal association of MS-k minimizing the objective function in problem P2 is as follows:

\[ x_{k,n} = \frac{r_{k,n}}{\lambda_k} \cdot \left[ \rho_{k,n} - \sqrt{1 + \gamma_k r_{k,n}/\lambda_k} \right]^+ \]

\( \gamma_k \) chosen such that \( \sum x_{k,n} = 1 \)
1. **Initialization:**
   Start from a feasible user association rule.

2. **Iterations:**
   Update the association pattern of each MS 1-by-1 with the rule in Proposition 2.

3. **Finalizing:**
   Make the converging association pattern a practical one:

\[ x^{(f)}_{k,n} = 1 \{ n | x^{(j)}_{k,n} > x^{(j)}_{k,l}, \forall l \neq n \} (n). \]

\( \mathcal{D} \): set of MSs with unbounded delays

\[
\begin{align*}
  n_k &= \arg \max_n \left\{ 1 - \sum_{k' \neq k} \frac{x^{(f)}_{k',n} \lambda_{k'}}{r_{k',n}} - \frac{\lambda_k}{r_{k,n}} \right\} \\
\text{repeat } k \in \mathcal{D} \\
\text{Update } x^{(f)}_{k,n} \text{ to } x^{(f)}_{k,n} = \delta(n - n_k) \text{ and } n_k \text{ denotes the BS with the maximum amount of available resources, i.e.} \\
\text{until termination test satisfied.}
\end{align*}
\]
QoS-aware Association-BCD

- QoSA-BCD is a distributed algorithm

Diagram:
- BS → MS-k
- ρ_{2,1}
- ρ_{2,2}
QoS-aware Association-BCD

- QoSA-BCD is a distributed algorithm

![Diagram showing a communication link between MS-k and BS](image-url)
QoSA-ADMM

- QoSA-BCD enjoys simplicity and can be readily applied on top of the conventional association schemes
- But conventional association schemes could result in unstable queues at the BS
  - A feasible association pattern is needed to start the QoSA-BCD
- QoSA-ADMM does not require a feasible start while still enjoying distributed implementation!
QoSA-ADMM

\[ \begin{align*}
\text{minimize} & \quad f(\{x_{k,n}\}) := \sum_{n=1}^{N} f_n(\{x_{k,n}\}) \\
\text{subject to} & \quad \sum_{n=1}^{N} x_{k,n} = 1, \quad \forall k = 1, \ldots, K \\
& \quad x_{k,n} \geq 0, \quad \forall k = 1, \ldots, K, \quad n = 1, \ldots, N \\
& \quad \sum_{k=1}^{K} \frac{x_{k,n} \hat{\lambda}_k}{r_{k,n}} < 1, \quad \forall n = 1, \ldots, N
\end{align*} \]

**P2**

\[ x_k := [x_{k,1}, \ldots, x_{k,N}]^T \]
\[ z_n := [z_{1,n}, \ldots, z_{K,n}]^T \]
\[ C_k := \{ x = [x_1, \ldots, x_N]^T | x \geq 0, \sum_{n=1}^{N} x_n = 1 \} \]
\[ D_n := \{ z = [z_1, \ldots, z_K]^T | z \geq 0, \sum_{k=1}^{K} z_k \hat{\lambda}_k / r_{k,n} \leq 1 \} \]
\[ \phi_k(x_k) = \mathbb{I}_{C_k}(x_k) = \begin{cases} 
0, & x_k \in C_k \\
\infty, & \text{o.w.}
\end{cases} \]
\[ \tilde{g}_n(z_n) = \frac{1}{1 - \sum_{k=1}^{K} z_{k,n} \frac{\hat{\lambda}_k}{r_{k,n}}} - \sum_{k=1}^{K} z_{k,n} \frac{\hat{\lambda}_k}{r_{k,n}} - 1 \]
Augmented Lagrangian for problem P3:

\[ L_\rho(x_{k,n}, z_{k,n}, \theta_{k,n}) = \sum_{k=1}^{K} \phi_k(x_k) + \sum_{n=1}^{N} \tilde{g}_n(z_n) + \sum_{k,n} \theta_{k,n}(x_{k,n} - z_{k,n}) + \frac{\rho}{2} \sum_{k,n} (x_{k,n} - z_{k,n})^2, \]

ADMM iterations are:

\[ x_{k}^{(j+1)} = \arg \min_{x_k} L_\rho(x_{k,n}, z_{k,n}^{(j)}, \theta_{k,n}^{(j)}), \]
\[ z_{n}^{(j+1)} = \arg \min_{z_n} L_\rho(x_{k,n}^{(j+1)}, z_{k,n}, \theta_{k,n}^{(j)}), \]
\[ \theta_{k,n}^{(j+1)} = \theta_{k,n}^{(j)} + \rho (x_{k,n}^{(j+1)} - z_{k,n}^{(j+1)}). \]
QoSA-ADMM

**ADMM iterations** can be implemented in a distributed manner to reach consensus:

**Each MS Update:**

\[
x^{(j+1)}_{k,n} = \left[ z^{(j)}_{k,n} - \frac{\theta^{(j)}_{k,n}}{\rho} - \frac{\nu}{\rho} \right]^+, \quad \text{where } \nu \text{ is chosen such that } \sum_{n=1}^{N} x^{(j+1)}_{k,n} = 1
\]

**Each BS Update:**

\[
z^{(j+1)}_{k,n} = \left[ x^{(j+1)}_{k,n} + \frac{\theta^{(j)}_{k,n}}{\rho} - \frac{1}{\rho} \frac{\lambda_{k}}{r_{k,n}} \cdot \left( \frac{1}{1 - \sum_{u=1}^{K} z^{(j+1)}_{u,n} \frac{\lambda_{u}}{r_{u,n}}} - 1 \right) \right]^+
\]

**Each BS Update:**

\[
\theta^{(j+1)}_{k,n} = \theta^{(j)}_{k,n} + \rho \left( x^{(j+1)}_{k,n} - z^{(j+1)}_{k,n} \right)
\]
Simulations

Simulated network layout with 5 macro-BSs (Red Triangle), 20 pico-BSs (Green Circle), and 400 MSs (Black Dot).
Simulations

CDFs of the average packet delay seen by each MS.
Conclusion

- For dense fog-like heterogeneous networks with QoS traffic, we have offered
  - Closed-form optimal resource allocation when users’ association pattern is fixed
  - Closed-form optimal association scheme for one MS when given others’ association
  - QoSA algorithms with different flavors:
    - minimize the average packet delay across the network
    - low complexity and distributed implementation
    - most desired feature in HetNet with lots of unplanned wireless nodes

Thanks for your attention!